Date _____ Week 4

Quadratic Equation

Subtopic: Roots of quadratic equation

Lesson Topic: sum and product of roots.

INSTRUCTIONAL RESOURCES: Graph board and graph papers

SPECIFIC OBJECTIVES:

Solving problems involving simultaneous equation

Solving problems involving quadratic equations

solving problems relating to sum and product of roots

Let's α and β denotes roots the roots of the quadratic equation:

$ax^2 + bx + c = 0$, where $a \neq 0$

$ax^2 + bx + c = 0$

using formular:

$\alpha + \beta = (-b)/a$, $\alpha\beta = c/a$

TEACHER ACTIVITIES: solution to factorization.

Suppose that

$ax2 + bx + c = (x - \alpha)(\alpha - \beta)$ then

$ax2 + bx + c = 0$, if $(x - \alpha)(\alpha - \beta) = 0$ since

$a \times b = 0$, is true only if either $a = 0$ or $b = 0$

(a, b are real numbers, then either $(x - \alpha) = 0$ or $(\alpha - \beta) = 0$ or both.

In general, suppose that there are integers p, q, r and s such that:

$ax^2 + bx + c = 0$, if $(p_x + q)(r_{x+} s)$

Find the quadratic equation the roots of which are the squares of the roots of the equation.

$ax^2 + bx + c = 0$

Solution:

$\alpha + \beta = (-b)/(a), \alpha\beta = c/(a)$

The roots are of the required equation on α^2 and β^2, we need to calculate the sum of the roots and product of roots.

$\alpha^2 + \beta^2$ and $\alpha^2\beta^2$

from symmetric identities,

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$(b\text{^}2\ )/a\text{^}2\ -\ 2c/(\text{a}) =\ (b\text{^}2 - 4ac\ )/a\text{^}2$ and

$\alpha^2\beta^2 = (\alpha\beta)^2 = (c\text{^}2\ )/a\text{^}2$

The required equation is:

$\text{x}2 - (\ (b\text{^}2 - 4ac\ )/a\text{^}2\ )x + (c\text{^}2\ )/a\text{^}2$

And on multiplying through by a^2

$a\text{^}2\ x^2 - (\ [\![2ac - b]\!]\ \text{^}2)x + c\text{^}2 = 0$

Example

The quadratic equation

$2x^2 - 3x + 6 = 0$

The two roots if α and β obtains a quadratic equation in x which has α^3 and β^3

Solution:

$\alpha + \beta = 3/2, \alpha\beta\ = 3$

therefore: $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha\ +\ \beta)$

by the symmetric identities

$= 27/8 - 3 \times 3(3/2)$

$=27/8 -\ \ 27/2 =\ -10\ 1/8$

$\alpha^3\beta^3 = (\alpha\beta)^3 = 27$

Solve:

Find the value of c in the quadratic equation:

$x^2 + \text{c}x + c = 0$

If one of the roots is half of the other

Solution:

Let the root be α and $1/(2\ )\ \alpha$ then

$3/(2\ )\ \alpha =\ -\text{c},\ \ 1/(2\ )\ \alpha\text{^}2\ \ = \text{c}$

Therefore: $\alpha\text{^}2 = 2c =\ \ (\ [\![4c]\!]\ \text{^}2\ )/9$

$c =\ \ (9\ )/2$

ASSIGNMENT:

If α and β are the roots of quadratic equations $2x^2 + 7x + 3 = 0$.

Obtain the equation the roots which are $(1\ )/(\alpha\text{^}2\ )$ and $(1\ )/(\beta\text{^}2\ )$

Let a b and c denotes real constant, show that if the quadratic equation:

$x^2 - (3c - b)x + bc = 0$, has equal roots, then so does:

$x^2 – (5c – b)x + 4c^2 = 0$